Intro

Solar radiation data is necessary for the design of solar heating systems and used to estimate the thermal performance of solar heating plants. Compared to global irradiance, the direct beam component shows much more variability in space and time. The global radiation split into beam and diffuse radiation on collector plane is important for the evaluation of the performance of different collector types and collector field designs. In the past, in most cases inexpensive and inaccurate solar radiation sensors were used to measure solar radiation on the collector planes in solar heating plants.

Generally, climate stations measure global radiation and only in rare cases DNI or diffuse solar radiation on the horizontal surface. Therefore, total irradiation on tilted surfaces in most cases is calculated by using measured global irradiation by means of empirical models for general use. Khorasanizadeh H., et al. set up a new diffuse solar radiation model to determine the optimum tilt angle of surfaces in Tabass, Iran [1]. Marques Filho, E., et al. carried out observational characterisation and empirical modelling of global, diffuse and direct solar radiation at surfaces in the city of Rio de Janeiro [2]. El Mghouchi, Y., et al. evaluated four empirical models to predict the daily direct diffuse and global radiations in Tutuan city, north of Morocco [3]. Jakhrani A. Q., et al. investigated the accuracy of different empirical models for calculations of total solar radiation on tilted surfaces [4]. It was found that the isotopic model (Liu and Jordan model) was better for prediction of solar energy radiation in cloudy conditions and could be used to calculate available solar radiation on tilted surfaces in overcast skies under Malaysian climate conditions. El-Sebaii, A. A., et al. also calculated diffuse radiation on horizontal surface and total solar radiation on tilted surfaces using empirical models [5]. They also found that the isotropic model (Liu and Jordan model) could be used to calculate total radiation on tilted surfaces with good accuracy in Jeddah, Saudi Arabia. Gopinathan K.K. investigated solar radiation on variously oriented sloping surfaces in Lesotho, South Africa with the isotropic model [6]. Li H., et al. estimated of daily global solar radiation in China [7]. Alyahya S., et al. analyzed the new solar radiation
Atlas for Saudi Arabia [8]. Bird R.E., et al. developed a simple solar spectral model for direct and diffuse irradiance on horizontal and tilted planes at the earth's surface for cloudless atmospheres [9]. There are also several studies on the prediction of solar radiation using machine learning and multivariable regression methods [10-11]. Jakhrani A.Q., et al. [12] and Despotovic M., et al. [13] investigated the accuracies of different empirical models in predicting total tilted solar radiation and diffuse horizontal solar radiation respectively. Ineichen P. came to the conclusion that the Perez model is slightly better (in terms of RMSD) than other models in any case, even with synthetic data [14]. Guemard C.A., et al. [15] carried out a comprehensive evaluation study of the performance of 140 separation models selected from the literature to predict direct normal irradiance from global horizontal irradiance. The evaluation was based on measured high-quality 1-min data of global horizontal irradiance and DNI at 54 research-class stations from 7 continents. Only two models consistently delivered the best predictions over the arid, temperate and tropical zones and no model performs consistently well over the high-albedo zone.

Using previous empirical models to convert global solar radiation data for general use in high latitude areas, such as Denmark, does not give highly accurate results [1-13]. Furthermore, limited literature was found on the analysis and prediction of total tilted solar radiation at high latitudes. A novel combined solar heating plant with a 4039 m$^2$ parabolic trough collector field and a 5960 m$^2$ flat plate collector field in Tårns was put into operation in August 2015 [16, 17]. To evaluate the thermal performance of the plant and accuracy of calculated solar radiation, total tilted and horizontal solar radiations were measured in the collector field. In addition, a weather station was in operation close to the solar collector fields to ensure that the pyranometers in the plant had correct values to reduce systemic errors and to measure the direct normal irradiance (DNI).

Calculation models of diffuse radiation on horizontal surface

Diffuse radiation influences the thermal performance of the flat plate collector field. The diffuse horizontal radiation was estimated by the RR model (Reduced Reindl correlation model) [18] and the DTU model [19]. The DTU model was developed based on measurements from 2006-2010 at a climate station at DTU [19] and was used to calculate diffuse radiation on horizontal surfaces with only global radiation as input. The RR model was developed by Reindl in 1990 for general use to calculate diffuse radiation on horizontal surfaces with only global radiation as input [20-21]. These two models are compared to the measured data from the Tårns plant.

When the diffuse radiation on horizontal surface has been calculated, the direct radiation on the same
surface can be derived by just subtraction of beam radiation from global radiation. DNI can then be determined by division by cosine of the zenith angle indirectly. The last two steps for direct radiation are exact numerical conversions without calculation error.

**Calculation models of total radiation for tilted surfaces**

Five calculation models for total radiation on tilted surfaces for general use are investigated: One isotropic model and four anisotropic models. Circumsolar diffuse and horizon-brightening components on the tilted surfaces have been taken into consideration in the anisotropic models, but not in the isotropic model.

The differences between monthly measured solar radiation and calculated solar radiation estimated by the empirical formulas from Aug.2015 to Sep.2016, including DNI, diffuse horizontal radiation and total titled solar radiation, were found. Mean bias error (MBE), root mean square error (RMSE), mean absolute percentage error (MAPE) and relative percentage error (RPE) were used to assess the feasibility of seven investigated empirical models. Calculated total tilted radiation based only on global radiation, and based on both global radiation and beam radiation were discussed and compared. A new method to calculate total tilted radiation based only on measured global radiation was suggested and maybe extended to other Nordic area that have similar weather.

**Models and Methodology**

**Data collection and location description**

As shown in figure 1, Denmark has 6 solar radiation zones with different yearly global radiations. The Tårn plant is located in the first solar radiation zone, in the northern part of Denmark. Figure 2 illustrates the locations of the weather station and the pyranometers in the flat plate collector field. The weather station is next to the solar heating plant. There are several pyranometers to measure global solar radiation and total radiation on the tilted plane of the flat plate collectors in the middle of flat plate collector field (figure 3). The latitude of Tårn is 57.39 °N and the longitude is 10.11°E respectively.
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Fig.1. Location of Tårn in Denmark [23-24]

Fig.2. Location of the weather station and pyranometers (PTC: parabolic trough collector, FPC: flat plate collector) [17]
Fig.3. The pyranometers in the middle of the flat plate collector field [29]

As shown in figure 2 and 3, four south facing pyranometers with a tilt 50° were installed on the top of a flat plate collector plane in the middle of the flat plate collector field. One is installed on horizontal surface. Two of the pyranometers to measure solar radiation on the horizontal surface and solar radiation on the titled collector plane were Kipp & Zonen SMP11, see Fig.3 left. DNI was measured with a PMO6-CC pyrheliometer with the sun tracking platform Sunscanner SC1 in the weather station next to the solar heating plant, see Fig.2 and 4. Table 1 and 2 show the technical specifications of Kipp & Zonen SMP11 pyranometer and PMO6-CC pyrheliometer [25-27]. It is estimated that the uncertainty of the measured solar radiation is about 2% based on the technical specifications.

Table 1. Specifications of Kipp & Zonen SMP11 Pyranometer [29]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spectral range (50% points)</td>
<td>285 to 2800 nm</td>
</tr>
<tr>
<td>Response time (63%)</td>
<td>&lt; 0.7 s</td>
</tr>
</tbody>
</table>
Response time (95%) < 2 s
Zero offset A < 7 W/m²
Zero offset B < 2 W/m²
Directional response (up to 80° with 1000 W/m² beam) < 10 W/m²
Temperature dependence of sensitivity (-20 °C to +50 °C) < 1 %
Analogue output (-V version) 0 to 1 V
Analogue output (-A version) 4 to 20 mA

Table 2. Specifications of PMO6-CC pyrheliometer [29]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimension</td>
<td>80 x 80 x 230 mm</td>
</tr>
<tr>
<td>Mass</td>
<td>2.15 kg</td>
</tr>
<tr>
<td>Field of view (full angle)</td>
<td>5°</td>
</tr>
<tr>
<td>Slope angle</td>
<td>1°</td>
</tr>
<tr>
<td>Range</td>
<td>up to 1400 W/m² (or custom design available)</td>
</tr>
<tr>
<td>Traceability to WRR</td>
<td>&lt; 0.1%</td>
</tr>
<tr>
<td>Operating temperature range</td>
<td>-25 °C to +50 °C</td>
</tr>
</tbody>
</table>

DNI, global radiation and total tilted solar radiation on the top of a 50° tilted south facing collector were measured with a high time resolution of 2 minutes. Hourly mean values were calculated based on the measured values. The calculated solar radiation was based on the mean data of one hour time step. Both the DTU model and the RR model were used to calculate diffuse radiation on the horizontal surface. Five other empirical models (one isotropic model and four anisotropic models) were used to calculate total solar radiation on the tilted surface. Ground reflectance or albedo was assumed to be 0.1. This value is a reasonable estimation of ground reflectance when shadows between collector rows in the solar heating plant are considered.

Measured horizontal diffuse radiation
Diffuse radiation on the horizontal surface was not measured directly in the Gårds plant. But diffuse radiation can be derived accurately as the difference between total radiation and beam radiation. Measured beam radiation was calculated by measured DNI and solar zenith angle by equation 1. Measured diffuse radiation on horizontal surface is determined by the difference between measured global radiation and the beam radiation component indirectly by equation 2.

\[ G_b = DNI \times \cos \theta \]  
(eq. 1)

\[ G_d \approx G - G_b \]  
(eq. 2)

**Modelled horizontal diffuse radiation**

1. **DTU model**

Dragsted et al. analyzed measured solar radiation from a climate station at the Technical University of Denmark from 2006 to 2010, and developed an empirical model to calculate horizontal diffuse radiation from global radiation on horizontal surface [19]. The empirical model is as follows, equation 3 - 7:

\[ K_r = \frac{G}{G_0} \]  
(eq. 3)

\[ G_d / G = -0.60921K_r^3 + 1.9982K_r^2 - 0.2787K_r + 1, \quad 0.00 \leq K_r < 0.29 \]  
(eq. 4)

\[ G_d / G = 3.99K_r^3 - 7.1469K_r^2 + 2.3996K_r + 0.746, \quad 0.29 \leq K_r < 0.72 \]  
(eq. 5)

\[ G_d / G = 288.63K_r^4 - 625.26K_r^3 + 448.06K_r^2 - 105.84K_r, \quad 0.72 \leq K_r < 0.80 \]  
(eq. 6)

\[ G_d / G = 65.89K_r^4 - 210.69K_r^3 + 222.91K_r^2 - 77.203K_r, \quad 0.80 \leq K_r < 1.20 \]  
(eq. 7)

2. **Reduced Reindl correlation model**

The Reduced Reindl correlation model is based on the relationships developed by Reindl et al.[20-21]. The Reduced Reindl model uses clearness index and solar altitude angle to estimate diffuse radiation on the horizontal surface. The correlation is given by the following equations 8-10:

\[ G_d / G = 1.020 - 0.254K_r + 0.0123\sin \alpha, 0 \leq K_r \leq 0.3, G_d / G \leq 1.0 \]  
(eq. 8)

\[ G_d / G = 1.400 - 1.794K_r + 0.177\sin \alpha, 0.3 \leq K_r \leq 0.78, 0.1 \leq G_d / G \leq 0.97 \]  
(eq. 9)

\[ G_d / G = 0.486K_r - 0.182\sin \alpha, 0.78 \leq K_r, 0.1 \leq G_d / G \]  
(eq. 10)

**Modelled total tilted solar radiation**
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(1) Isotropic model
The typical isotropic model was developed by Liu and Jordan (Liu-Jordan model, Equation 11) and used widely. The isotropic model assumes that diffuse radiation is uniformly distributed over the complete sky dome.

\[ G_r = G_b R_b + G_d \left( \frac{1 + \cos \beta}{2} \right) + G \rho_g \left( \frac{1 - \cos \beta}{2} \right) \]  \hspace{1cm} (eq. 11)

(2) Anisotropic model
1. Hay and Davies model-(HD model)
The Hay and Davies model accounts for both circumsolar and isotropic diffuse radiation. There is an increased intensity of diffuse radiation in the area around the sun (circumsolar diffuse radiation). An anisotropy index is introduced in the HD model to weight the amount of circumsolar diffuse radiation.

\[ A_i = DNI / G_b \]  \hspace{1cm} (eq. 12)

\[ G_r = (G_b + G_d A_i) R_b + G_d (1 - A_i) \left( \frac{1 + \cos \beta}{2} \right) + G \rho_g \left( \frac{1 - \cos \beta}{2} \right) \]  \hspace{1cm} (eq. 13)

2. HDKR model
A horizon brightening diffuse term was added to the HD model by Reindl et.al. in the HDKR model[22]. The horizon brightening is lumped with the isotropic diffuse term and the magnitude named by a modulating factor \( \sqrt{\frac{G_d}{G}} \).

\[ G_r = (G_b + G_d A_i) R_b + G_d (1 - A_i) \left( \frac{1 + \cos \beta}{2} \right) \left( 1 + \frac{G_d}{G} \sin^3 \left( \frac{\beta}{2} \right) \right) + G \rho_g \left( \frac{1 - \cos \beta}{2} \right) \]  \hspace{1cm} (eq. 14)

3. Perez I model
Perez et.al. [20-22]developed the model accounting for circumsolar, horizon brightening and isotropic diffuse radiation by empirically derived “reduced brightness coefficient”. This is called Perez I model.

\[ G_r = G_b R_b + G_d \left[ (1 - F_1) \left( \frac{1 + \cos \beta}{2} \right) + F_1 \left( \frac{a}{c} \right) + F_2 \sin \beta \right] + G \rho_g \left( \frac{1 - \cos \beta}{2} \right) \]  \hspace{1cm} (eq. 15)

4. Perez II model
The Perez II model has the same formulation as the Perez I model. Both models differ only in the \( F_1 \) and \( F_2 \) coefficients. The method of calculating the detailed parameters \( a, c, F_1 \) and \( F_2 \) in the Perez I and Perez II model
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can be found in [20-22].

Methodology

Fig. 5 shows the schematic illustration of this study. Firstly, the DTU model and RR model are used to calculate horizontal diffuse radiation. Measured global horizontal radiation, DNI and measured total tilted radiation.
radiation were used to evaluate the suitability of the empirical models for total tilted radiation for Danish conditions (Fig.5a. Validation cycle). Then the selected empirical models based on calculated diffuse radiation and beam radiation were employed to calculate total tilted radiation (Fig.5b.). Calculated total tilted radiation with the DTU model and the investigated empirical models only based on global radiation shows good agreement with measured values from Sep.2015 to Aug.2016 (Fig.5b.). DNI calculated by the DTU model also has a good agreement with measured DNI. To sum up, the proposed method to calculate total tilted solar radiation only based on measured global horizontal radiation (Red flow chart) is a new simple and cost-effective approach to obtain accurate total tilted solar radiation for Danish conditions as measured global radiation is always available from climate stations. Furthermore, DNI and diffuse radiation measurements are relatively costly both in equipment and manpower. Accurate long-term data of these variables is seldom available in most cases. Therefore, accurately calculated DNI, diffuse radiation and total tilted radiation only based on measured global radiation are very valuable.

**Measured and calculated solar radiations**

**Diffuse radiation on the horizontal surface**

![Diffuse radiation on the horizontal surface](image)

Fig.6. Calculated and measured diffuse radiation on horizontal surface (Sep.2015 – Aug.2016) [29]

Measured diffuse radiation and calculated diffuse radiation based on the DTU model and RR model only using measured global radiation are shown in Figure 6. Monthly calculated results by the RR model are 6% lower than the measured values on average in Tårn. Diffuse radiation calculated by the DTU model is closer...
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to measured values than the RR model. The difference between the measured and simulated diffuse radiation by the DTU model is about 3% on average. The DTU model was developed for calculation of diffuse radiation at a DTU weather station in the fifth solar radiation zone, while the solar radiation measurements at Tårs took place in first solar radiation zone. It can be concluded that the DTU model is suitable for different solar radiation zones in Denmark.

**Total radiation on the tilted surface**

Calculated total radiation on the tilted surface by use of the isotropic and anisotropic models based on measured total horizontal radiation and measured beam radiation from Sep.2015 to Aug.2016 are shown in figure 7 together with measured values. The surface is facing south with a tilt of 50°.

![Fig.7. Calculated and measured total radiation on 50° tilted south facing surface (Sep.2015 – Aug.2016) [29]](image)

The calculated total tilted radiation by the isotropic model is quite lower than the measured values. Contrary to the conclusions under Saudi Arabia and Malaysia weather conditions in the references [3-4], the anisotropic models are better than the isotropic model under Danish climate conditions. For the four anisotropic models, the calculated total tilted radiation of the Perez II model and the Perez I model gives results closest to the measured values with average differences of only 1-2%, which is similar to what reported by Andersen E., et al. [28].

**Comparison of the different models**
Measured data from the Tårs solar heating plant were used to evaluate the models. Four statistical error parameters were introduced to evaluate the monthly results from Sep.2015 to Aug.2016.

(1). MBE, mean bias error

\[ MBE = \frac{1}{k} \sum_{i=1}^{k} \left( G_{\text{Calculated}}^i - G_{\text{Measured}}^i \right) \] (eq. 15)

(2). RMSE, root mean square error

\[ RMSE = \left( \frac{1}{k} \sum_{i=1}^{k} \left( G_{\text{Calculated}}^i - G_{\text{Measured}}^i \right)^2 \right)^{1/2} \] (eq. 16)

(3). MAPE, mean absolute percentage error

\[ MAPE = \frac{100}{k} \sum_{i=1}^{k} \left| \frac{G_{\text{Calculated}}^i - G_{\text{Measured}}^i}{G_{\text{Measured}}^i} \right| \] (eq. 17)

(4). RPE, relative percentage error

\[ RPE = \frac{1}{\sum_{i=1}^{k} G_{\text{Measured}}^i} \sum_{i=1}^{k} \left( G_{\text{Calculated}}^i - G_{\text{Measured}}^i \right) \] (eq. 18)

Comparisons between measured and calculated values of diffuse radiation on horizontal surface and total radiation on the tilted surface are shown in table 3 and 4 respectively [29]. The lower the MBE and RMSE are, the better the agreement between measured and calculated values is. For MBE, a positive value means an overestimation of the calculated values and a negative MBE means an underestimation of the calculated values. A drawback of MBE is that one positive value in one calculation step may cancel a negative value in another calculation step. RMSE is always positive. MAPE is positive and a low MAPE means the model is accurate. A negative RPE means the proposed model slightly underestimates the radiation. Table 3 shows that the DTU model is more accurate than the RR model for Danish conditions. Table 4 illustrates that the anisotropic models (Perez II model and Perez I model) are the most accurate among the investigated empirical models and best suitable for calculations of total tilted radiation under Danish conditions.

<table>
<thead>
<tr>
<th>Table 3 Measured and calculated MBE (kWh/m²), RMSE (kWh/m²), MAPE (-) and RPE (%) for diffuse horizontal radiation [29]</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTU model</td>
</tr>
<tr>
<td>MBE</td>
</tr>
</tbody>
</table>


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<table>
<thead>
<tr>
<th></th>
<th>Perez II model</th>
<th>Perez I model</th>
<th>HDKR model</th>
<th>HD model</th>
<th>Isotropic model</th>
</tr>
</thead>
<tbody>
<tr>
<td>MBE</td>
<td>-2.4</td>
<td>-3.4</td>
<td>-8.4</td>
<td>-10.0</td>
<td>-18.6</td>
</tr>
<tr>
<td>RMSE</td>
<td>2.0</td>
<td>2.6</td>
<td>4.9</td>
<td>5.8</td>
<td>10.0</td>
</tr>
<tr>
<td>MAPE</td>
<td>2.1</td>
<td>2.8</td>
<td>5.7</td>
<td>5.9</td>
<td>12.0</td>
</tr>
<tr>
<td>RPE</td>
<td>-1.2%</td>
<td>-1.8%</td>
<td>-4.3%</td>
<td>-5.2%</td>
<td>-9.7%</td>
</tr>
</tbody>
</table>

Table 4 Measured and calculated MBE (kWh/m²), RMSE (kWh/m²), MAPE (-) and RPE (%) for monthly total tilted radiation [29]

Measured and calculated DNI

Global radiation is always available from the climate stations of the Danish Meteorological Institute (DMI). DNI is not measured at climate stations and only seldom in solar heating plants. Moreover, DNI is a very important design parameter for concentrating collectors, such as the parabolic trough collectors in Tårn. Diffuse radiation calculated by the DTU model is more accurate than the RR empirical model under Danish conditions. So the DTU model was used in this section to predict DNI. By equation 1 and 2, the diffuse radiation calculated by the DTU model was used to calculate DNI or beam radiation. Figure 8 shows monthly calculated DNI (DTU) and measured DNI from Sep.2015 to Aug.2016 [29]. The calculated total DNI (983 kWh/m²) is about 2% larger than measured total DNI (963kWh/m²) for the period from Sep.2015 to Aug.2016, which is within the measuring accuracy.
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Fig. 8. Calculated DNI (DTU) and measured DNI in the Tårn solar heating plant. [29]

Measured and calculated total tilted radiation

As mentioned, normally global radiation from the Danish Meteorological Institute is available. Total radiation on collector surfaces is measured at most solar heating plants but with a poor accuracy. By the DTU model and equation 1 and 2, calculated diffuse radiation and beam radiation could be obtained only based on measured global radiation on horizontal surface. In addition, because the isotropic model could be used easily and widely and the anisotropic model (Perez II model and Perez I model) are closest to the measured values as shown in the previous sections, the isotropic model and the anisotropic models (Perez II and Perez I) were selected to calculate total radiation on the tilted surface based on calculated diffuse radiation and calculated beam radiation, which is calculated only from measured global radiation. The calculated total radiation on the tilted surface by the isotropic model is 8% lower than the measured one from Sep.2015 to Aug.2016. The calculated total tilted radiation by the Perez I model and Perez II model is less than 1% different from the total measured radiation (Fig.9.a-c.). Both of the Perez models have the best agreement with measurements.
Fig. 9. Measured monthly tilted total radiation and calculated tilted total radiation based on calculated...
diffuse radiation and beam radiation (Sep.2015 – Aug.2016: a-isotropic model, b-Perez I model, c- Perez II model.) [29]

These results are in good agreement with the conclusions presented by Andersen E., et al. [28]. From the above results, it was found that the DTU model together with the Perez II and I models could be used to predict total radiation on tilted surfaces only based on measured global radiation under Danish conditions. Furthermore, the proposed models can be employed to check measured total radiations on tilted flat plate collector planes in solar heating plants in Denmark. The proposed method can also be employed to derive solar radiation data for planning solar collector fields based on available horizontal global radiation measurements.

Radiation modelling for flat plate collector surfaces within a collector array

The analysed isotropic model and anisotropic models (HD, HDKR, Perez I and II) calculate the direct and diffuse irradiance for a tilted surface. Solar collector fields with flat plate collectors often have the same tilt and azimuth for all collectors of the array. For the tilted surface of the collectors in the front row, the models are applicable as they are for conditions without external shading if the horizon is even and the ground in front of the collectors is flat and has the same reflectance coefficient. Under these conditions, each surface segment of the collectors in the front row has the same view factor towards the sky and the same view factor towards the ground. For collectors within an array internal shading of the beam radiation occurs (Fig. 10).

Fig. 10. Internal shading for collectors within an array [30]
Besides internal shading, the obstruction of the front row collectors leads to the following effects:

- Collectors receive less diffuse radiation from the sky, because the sky view factor is smaller.
- Collectors receive less diffuse radiation from the ground, because the ground view factor is smaller.
- The ground receives less radiation from the sky, due to ground shading and a smaller sky view factor diminishing the diffuse radiation.
- The collector receives additional radiation from the backside of the collector placed in front. This effect is typically very small and can be neglected.

If the row spacing is narrow and the collector tilt is steep, radiation models for the tilted surface should be adapted in order to get a more precise estimation of the received irradiance on the collectors. An approach to improve the radiation modelling for collectors within the field is to divide the collector and ground in segments and calculate the received beam and diffuse irradiance separately for each segment. Fig. 10 shows a division in 20 collector segments of equal length (C₁ to C₂₀) and 20 ground segments (G₁ to G₂₀) of equal length.

![Fig. 10. Division of the collector and ground in segments](image)

The model assumes that the length of the collector row is much larger than the height, i.e. the collector row is of infinite length. This allows to calculate the view factor between two surfaces according to Fig. 11, using the ‘Hottel’s crossed-string rule’ [31].
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Fig. 11. Calculation of the view factor for two surfaces of infinite length

The formula is shown in (eq. 20), where \( F_{1 \rightarrow 2} \) is the view factor from surface \( A_1 \) to \( A_2 \).

\[
F_{1 \rightarrow 2} = \frac{CF+DE-CE-DF}{2 \cdot CD} \quad \text{(eq. 20)}
\]

How the radiation models can be adapted is shown below for the isotropic model. Eq. 11 is changed to the following form based on \( m \) collector segments and \( n \) ground segments

\[
G_T = G_b R_b S + G_d \cdot \frac{1}{m} \sum_{i=1}^{m} F_{i \rightarrow sky} + \frac{1}{n} \sum_{i=1}^{m} \sum_{j=1}^{n} \left( G_b \rho_g S_j + G_d \rho_g F_{j \rightarrow sky} \right) \cdot F_{i \rightarrow j} \quad \text{(eq. 21)}
\]

where \( S \) is the share of the collector surface which is not shaded from the beam (circumsolar) irradiance and \( S_j \) is the unshaded share for the \( j \)-th ground segment. The essential extension in comparison to eq. 11 is that the view factors \((F_{i \rightarrow sky}, F_{j \rightarrow sky}, F_{i \rightarrow j})\) are calculated for each segment individually, using eq. 20. Furthermore, the received irradiance for the ground is split in a beam (circumsolar) and diffuse part. Internal and ground shading is considered with shading factors. The shading factor can be calculated based on [32].

The average irradiance on the collectors \( G_T \) is smaller than the irradiance on the collectors in the front row which are not obstructed. Fig. 12 shows exemplary results of the radiation reduction along the collector height for the collector array geometry depicted in Fig. 10 with (1) row spacing = 4 m, (2) collector height = 2.5 m, (3) collector tilt angle = 30°, (4) reflectance coefficient = 0.2. The calculation was done for a beam irradiance of 700 W/m\(^2\) and diffuse irradiance of 300 W/m\(^2\) on the horizontal surface. The columns named \( C_1 \) to \( C_{20} \) show the calculated irradiance for each of the 20 collector segments, starting from the bottom of the collector. The column to the right is the irradiance on the collector in the front row. As there is no shading, the beam irradiance is the same for all segments of the obstructed collector and the front collector. The diffuse irradiance however, is significantly less for the segments towards the ground, with an average of 257 W/m\(^2\) for the collector within the field and 293 W/m\(^2\) for the collector in the front row. In this example,
the row spacing is narrow, which leads to a strong effect compared to a wider row spacing, where the difference is usually small.

Fig. 12. Irradiance along the collector height (left columns) and for front collector (right column)

Conclusions

Measured and calculated monthly horizontal diffuse solar radiation and total tilted solar radiation from September 2015 to August 2016 in a demonstration solar heating plant in Denmark were analyzed. The DTU model, developed for calculation of horizontal diffuse radiation in the solar radiation zone 5 of Denmark, was evaluated by the measured data in Tårns (Zone 1). It can be concluded that the DTU model is suitable for Danish conditions. Furthermore, an isotropic model and four anisotropic models for general use have been investigated for calculation of total monthly radiation on tilted surfaces under Danish climate conditions. Calculated monthly DNI based on the DTU model with only measured global radiation as input was also investigated with good agreement between calculations and measurements.

It is concluded that the DTU model can be used for calculation of diffuse radiation on the horizontal surface in Denmark. Anisotropic models can be used to calculate total radiation on tilted surfaces with better accuracy than the isotropic model under Danish conditions. Anisotropic models together with the DTU model
can be a new method to determine total radiations on tilted surfaces for Danish conditions. The only input for the mentioned method is global radiation measurement. The proposed method is very simple, cost-effective and gives relatively accurate total tilted radiation and DNI under Danish conditions.

Nomenclature

\( \beta \)  Incidence angle, °

\( \theta \)  Zenith angle, °

\( \alpha \)  Solar altitude angle, °

\( R_b \)  The ratio of beam radiation on the tilted surface to that on a horizontal surface at any time

\( \rho_d \)  Diffuse reflectance for the total solar radiation

\( A \)  Anisotropy index

\( k \)  Number of calculated values

\( i \)  Every calculated value

\( G \)  Mean total radiation on the horizontal surface, W/m²

\( G_d \)  Mean diffuse radiation on the horizontal surface, W/m²

\( G_0 \)  Mean extraterrestrial radiation on a horizontal surface, W/m²

\( G_T \)  Mean total radiation on the tilted surface, W/m²

\( G_b \)  Mean beam radiation on horizontal surface, W/m²

\( K_T \)  Clearness index

\( G_{\text{Calculated}} \)  Calculated solar radiation, kWh/m²

\( G_{\text{Measured}} \)  Measured solar radiation, kWh/m²

\( \text{DNI} \)  Direct normal irradiance, W/m²

\( \text{MBE} \)  Mean bias error, kWh/m²

\( \text{RMSE} \)  Root mean square error, kWh/m²

\( \text{MAPE} \)  Mean absolute percentage error, -

\( \text{RPE} \)  Relative percentage error, -

\( \text{DTU} \)  Technical University of Denmark

\( \text{RR model} \)  Reduced Reindl correlation model

\( \text{PTC} \)  parabolic trough collector

\( \text{FPC} \)  flat plate collector

References
C-D1. Simulation and design of collector array units within large systems


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